

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Silver Level S2

Time: 1 hour 30 minutes

Materials required for examination
papers

Mathematical Formulae (Green)

Items included with question

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
69	62	55	46	43	39

1.

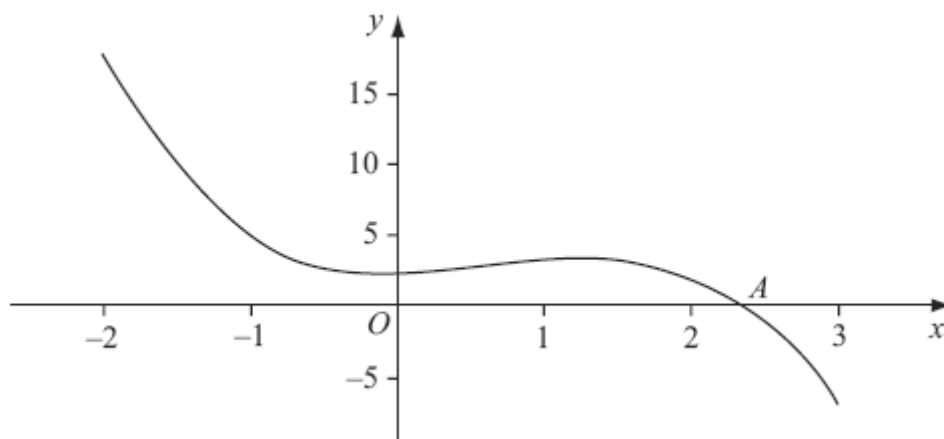


Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x -axis at the point A where $x = \alpha$.

To find an approximation to α , the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1 , x_2 , x_3 and x_4 .

Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)

June 2009

2. $f(x) = x^3 + 3x^2 + 4x - 12$
- (a) Show that the equation $f(x) = 0$ can be written as
- $$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \quad x \neq -3. \quad (3)$$

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

- (b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \quad n \geq 0,$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1 , x_2 and x_3 . (3)

The root of $f(x) = 0$ is α .

- (c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places. (3)

June 2012

3. $f(x) = 2 \sin(x^2) + x - 2, \quad 0 \leq x < 2\pi.$
- (a) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$. (2)

The equation $f(x) = 0$ can be written as $x = [\arcsin(1 - 0.5x)]^{\frac{1}{2}}.$

- (b) Use the iterative formula

$$x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}, \quad x_0 = 0.8,$$

to find the values of x_1 , x_2 and x_3 , giving your answers to 5 decimal places. (3)

- (c) Show that $\alpha = 0.80157$ is correct to 5 decimal places. (3)

June 2011

4. $f(x) = \ln(x+2) - x + 1, \quad x > -2, x \in \mathbb{R}.$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5,$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

January 2008

5. The function f is defined by

$$f: x \mapsto \frac{2(x-1)}{x^2 - 2x - 3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \frac{1}{x+1}, \quad x > 3$.

(4)

(b) Find the range of f .

(2)

(c) Find $f^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$g: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \frac{1}{8}$.

(3)

June 2008

6. The functions f and g are defined by

$$f: x \mapsto \ln(2x - 1), \quad x \in \mathbb{R}, \quad x > \frac{1}{2},$$

$$g: x \mapsto \frac{2}{x-3}, \quad x \in \mathbb{R}, \quad x \neq 3.$$

- (a) Find the exact value of $fg(4)$. (2)
- (b) Find the inverse function $f^{-1}(x)$, stating its domain. (4)
- (c) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y -axis. (3)
- (d) Find the exact values of x for which $\left| \frac{2}{x-3} \right| = 3$. (3)

June 2007

7.

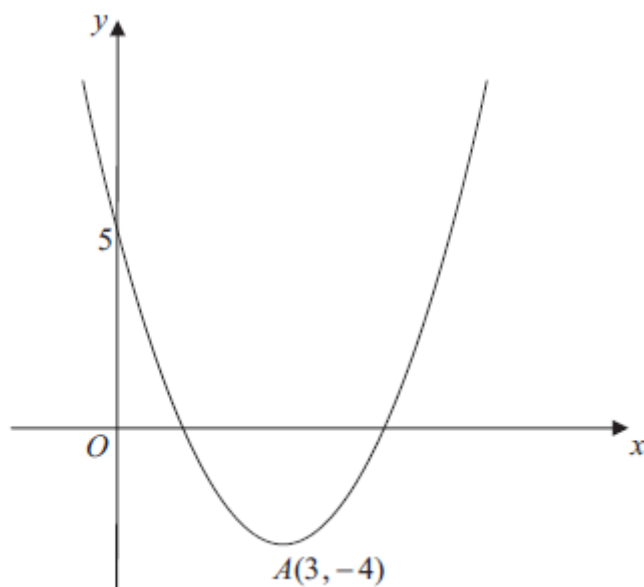


Figure 2

Figure 2 shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f(\frac{1}{2}x)$.

(4)

(b) Sketch the curve with equation $y = f(|x|)$.

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y -axis.

(3)

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $f(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

June 2010

8. (a) Differentiate with respect to x ,

(i) $x^{\frac{1}{2}} \ln(3x)$,

(ii) $\frac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3 \tan 2y$ find $\frac{dy}{dx}$ in terms of x .

(5)

June 2012

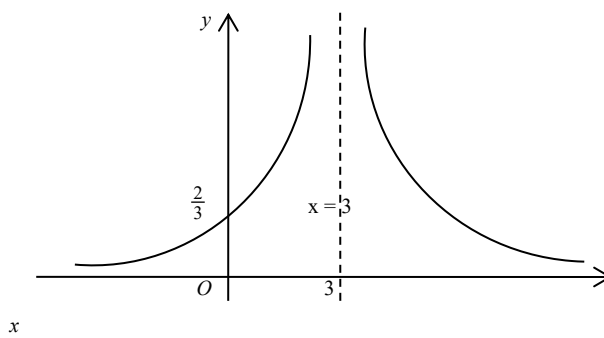
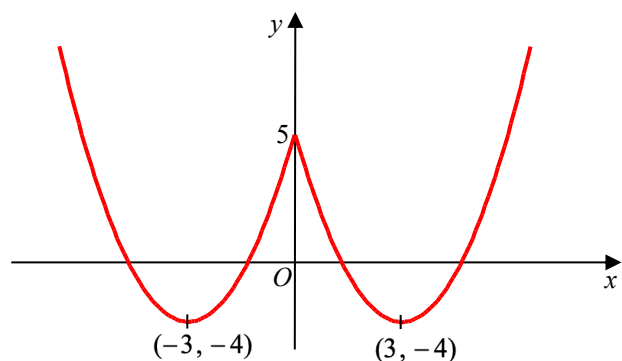
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$ $x_1 = \frac{2}{(2.5)^2} + 2$ $x_1 = 2.32$, $x_2 = 2.371581451...$ $x_3 = 2.355593575...$, $x_4 = 2.360436923...$	M1 A1 A1 cso (3)
(b)	Let $f(x) = -x^3 + 2x^2 + 2 = 0$ $f(2.3585) = 0.00583577...$ $f(2.3595) = -0.00142286...$ Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	M1 M1 A1 (3) [6]
2. (a)	$x^3 + 3x^2 + 4x - 12 = 0 \Rightarrow x^3 + 3x^2 = 12 - 4x$ $\Rightarrow x^2(x + 3) = 12 - 4x$ $\Rightarrow x^2 = \frac{12 - 4x}{(x + 3)} \Rightarrow x = \sqrt{\frac{4(3 - x)}{(x + 3)}}$	M1 dM1A1* (3)
(b)	$x_1 = 1.41$, awrt $x_2 = 1.20$ $x_3 = 1.31$	M1A1, A1 (3)
(c)	Choosing (1.2715, 1.2725) or tighter containing root 1.271998323 $f(1.2725) = (+)0.00827...$ $f(1.2715) = -0.00821....$ Change of sign $\Rightarrow \alpha = 1.272$	M1 M1 A1 (3) [9]

Question Number	Scheme	Marks
3. (a)	$f(0.75) = -0.18\dots$ $f(0.85) = 0.17\dots$	M1
	Change of sign, hence root between $x = 0.75$ and $x = 0.85$	A1
		(2)
	(b) Sub $x_0 = 0.8$ into $x_{n+1} = \left[\arcsin(1 - 0.5x_n) \right]^{\frac{1}{2}}$ to obtain x_1	M1
(c)	Awrt $x_1 = 0.80219$ and $x_2 = 0.80133$	A1
	Awrt $x_3 = 0.80167$	A1
	$f(0.801565) = -2.7\dots \times 10^{-5}$ $f(0.801575) = +8.6\dots \times 10^{-6}$	A1
	Change of sign and conclusion	A1
		(3)
		[8]
4.	(a) $f(2) = 0.38 \dots$ $f(3) = -0.39 \dots$	M1
	Change of sign (and continuity) \Rightarrow root in $(2, 3)$ *	cso A1
		(2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$ $x_2 \approx 2.50498$ $x_3 \approx 2.50518$	M1
(c)		A1
		A1
		(3)
	Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends.	M1
	$f(2.5045) \approx 6 \times 10^{-4}$ $f(2.5055) \approx -2 \times 10^{-4}$	
	Change of sign (and continuity) \Rightarrow root $\in (2.5045, 2.5055)$	
	\Rightarrow root = 2.505 to 3 dp *	cso A1
		(2)
		[7]

Question Number	Scheme	Marks
5. (a)	$x^2 - 2x - 3 = (x-3)(x+1)$ $f(x) = \frac{2(x-1)-(x+1)}{(x-3)(x+1)} \left(\text{or } \frac{2(x-1)}{(x-3)(x+1)} - \frac{x+1}{(x-3)(x+1)} \right)$ $= \frac{x-3}{(x-3)(x+1)} = \frac{1}{x+1} *$	B1 M1 A1 A1 cso (4)
(b)	$\left(0, \frac{1}{4}\right)$ Accept $0 < y < \frac{1}{4}$, $0 < f(x) < \frac{1}{4}$ etc.	B1 B1 (2)
(c)	Let $y = f(x)$ $y = \frac{1}{x+1}$ $x = \frac{1}{y+1}$ $yx + x = 1$ $y = \frac{1-x}{x}$ or $\frac{1}{x} - 1$ $f^{-1}(x) = \frac{1-x}{x}$ Domain of f^{-1} is $\left(0, \frac{1}{4}\right)$	M1 A1 B1 ft (3)
(d)	$fg(x) = \frac{1}{2x^2 - 3 + 1}$ $\frac{1}{2x^2 - 2} = \frac{1}{8}$ $x^2 = 5$ $x = \pm\sqrt{5}$	M1 A1 both A1 (3)
		[12]

Question Number	Scheme	Marks	
6. (a)	Finding $g(4) = k$ and $f(k) = \dots$ or $fg(x) = \ln\left(\frac{4}{x-3} - 1\right)$	M1	
	$[f(2) = \ln(2 \times 2 - 1) \quad fg(4) = \ln(4 - 1)] = \ln 3$	A1 (2)	
	$y = \ln(2x - 1) \Rightarrow e^y = 2x - 1$ or $e^x = 2y - 1$ $f^{-1}(x) = \frac{1}{2}(e^x + 1)$ Allow $y = \frac{1}{2}(e^x + 1)$ Domain $x \in \mathbb{R}$ [Allow \mathbb{R} , all reals, $(-\infty, \infty)$] independent	M1, A1 A1 B1 (4)	
		Shape, and x-axis should appear to be asymptote Equation $x = 3$ needed , may see in diagram (ignore others) Intercept $(0, \frac{2}{3})$ no other; accept $y = \frac{2}{3}$ (0.67) or on graph	B1 B1 ind. B1 ind (3)
	$\frac{2}{x-3} = 3 \Rightarrow x = 3\frac{2}{3}$ or exact equiv. $\frac{2}{x-3} = -3, \Rightarrow x = 2\frac{1}{3}$ or exact equiv.	B1 M1, A1 (3) [12]	
7. (a)(i) (ii) (b) (c) (d)	(3, 4) (6, -8)	B1 B1 B1 B1 (4)	
		B1 B1 B1	
	$f(x) = (x - 3)^2 - 4$ or $f(x) = x^2 - 6x + 5$	(3) M1A1 (2)	
	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1 (1) [10]	

Question Number	Scheme	Marks
8. (a)	(i) $\frac{d}{dx}(\ln(3x)) = \frac{3}{3x}$	M1
	$\frac{d}{dx}(x^{\frac{1}{2}} \ln(3x)) = \ln(3x) \times \frac{1}{2} x^{-\frac{1}{2}} + x^{\frac{1}{2}} \times \frac{3}{3x}$	M1A1
		(3)
	(ii) $\frac{dy}{dx} = \frac{(2x-1)^5 \times -10 - (1-10x) \times 5(2x-1)^4 \times 2}{(2x-1)^{10}}$	M1A1
	$\frac{dy}{dx} = \frac{80x}{(2x-1)^6}$	A1
(b)		(3)
	$x = 3 \tan 2y \Rightarrow \frac{dx}{dy} = 6 \sec^2 2y$	M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6 \sec^2 2y}$	M1
	Uses $\sec^2 2y = 1 + \tan^2 2y$ and uses $\tan 2y = \frac{x}{3}$	
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6(1+(\frac{x}{3})^2)} = (\frac{3}{18+2x^2})$	M1A1
		(5)
		[11]

Examiner reports

Question 1

This question was generally well answered with many successful attempts seen in both parts. There were few very poor or non-attempts at this question.

In part (a), the majority of candidates were able to score all three marks. There were a significant number of candidates in this part who incorrectly gave x_3 and x_4 as 2.355 and 2.361 respectively. These incorrect answers were usually achieved by candidates substituting the rounded answer of $x_2 = 2.372$ to find x_3 and substituting their rounded answer of $x_3 = 2.355$ to find x_4 . Some candidates are not aware that it is possible to program a basic calculator by using the ANS button to find or even check all four answers. Another common error in this part was for candidates to stop after evaluating x_3 .

The majority of candidates who attempted part (b) choose an appropriate interval for x and evaluated y at both ends of that interval. The majority of these candidates chose the interval (2.3585, 2.3595) although incorrect intervals, such as (2.358, 2.360) were seen. There were a few candidates who chose the interval (2.3585, 2.3594). This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the correct result and this was accepted for full marks. To gain the final mark, candidates are expected to give a reason that there is a sign change, and give a suitable conclusion such as that the root is 2.359 to 3 decimal places or $\alpha = 2.359$ or even QED.

A minority of candidates attempted part (b) by using a repeated iteration technique. Almost all of these candidates iterated as far as x_6 (or beyond) but most of these did not write down their answers to at least four decimal places. Of those candidates who did, very few candidates managed to give a valid conclusion.

Question 2

This was completed very well with many candidates achieving full marks. In part (a) most candidates managed to rearrange the formula to $x^2(x + 3) = (12 - 4x)$ and, when they got to this stage, generally managed to proceed to the correct answer. Common mistakes included not factorising out x^2 before dividing by $(x + 3)$, and notation errors in which the square root appeared on only the numerator of the fraction. Incorrect methods usually started when the candidates put just 12 on one side of the equation, and factorised the other therefore rendering a correct result impossible. Those candidates who opted for working backwards did not usually state $f(x) = 0$ at the end of their proof. Attempting to divide $f(x)$ by $x + 3$ was rarely seen, but hardly ever completed correctly.

Part (b), was well answered with a small minority of candidates leaving their answer as root 2 for x_1 . A few did make errors in their calculations but these were in the minority. Almost all attempted this part.

Part (c) was familiar to students and there were many fully correct solutions. Although this type of question has been asked in many sessions a number of candidates did not give either a valid reason as well as a valid conclusion.

Question 3

Part (a) was generally very well answered and nearly all correctly calculated $f(0.75)$ and $f(0.85)$. Most candidates now seem to realise that they need to give both reason and

conclusion to justify their answer, although a few failed to do this, often just writing ‘change of sign’. There were a few who had calculators set in degree mode, and some made careless errors, but these were in the minority of cases.

Part (b) was also more often than not fully correct. There was some evidence however of candidates who were not familiar with the arcsin function. As well as some who wrote “no arcsin button” there were other common errors which included

- (i) evaluating $\sqrt{1 - 0.5x_n}$ and ignoring the arcsin,
- (ii) $\arcsin \sqrt{1 - 0.5x_n}$,
- (iii) $\frac{1}{\sin(1 - 0.5x_n)}$ (with or without the square root),
- (iv) $\sqrt{\sin(1 - 0.5x_n)}$,
- (v) working in degrees.

Very few candidates failed to give the required accuracy, or rounded incorrectly.

In part (c) the majority chose the correct interval $[0.801565, 0.801575]$ and completed the proof correctly. There were only a few incorrect intervals. Candidates who chose to proceed by repeated iteration were less successful. Most of them only worked to 5 decimal places and many did not proceed as far as x_7 . Of the few who gave all the necessary results, there were not many who also gave a sufficient conclusion to complete the proof.

Question 4

In parts (a) and (c), candidates need to be aware that showing that something is true requires them to give reasons and conclusions. In this part (a), it is sufficient to say that a change of sign in the interval (2, 3) implies that there is a root in the interval (2, 3). In part (c), it would be sufficient to argue that a change of sign in the interval (2.5045, 2.5055) implies that there is a root in the interval (2.5045, 2.5055) and, hence, that $x = 2.505$ is accurate to 3 decimal places.

In part (a), most candidates chose the obvious 2 and 3 and successfully found $f(2)$ and $f(3)$ to gain the method mark.

The majority of candidates now seem comfortable with the method of iteration. Part (b) was particularly well answered with only a minority of candidates making errors, mainly over issues of accuracy.

In part (c) candidates who chose (2.5045, 2.5055) were more often successful than not. Although it is not a wholly satisfactory method, on this occasion the examiners did accept repeated iteration. The candidates were required to reach at least $6x$, showing their working to 5 decimal places, which most choosing this method did, and to give a reason why they concluded that the root was accurate to 3 decimal places. This second requirement was rarely met.

Question 5

The method of simplifying fractions was better known than in some previous examinations with many fully correct solutions. As noted in the introduction above, a failure to use brackets can lead to a loss of marks. Candidates should not assume that examiners will read $2(x - 1) - x + 1$ as $2(x - 1) - (x + 1)$. Part (b) was one of the most testing parts of the paper

and very few obtained the fully correct answer $0 < f(x) < \frac{1}{4}$. Many had no idea at all how to find the range and many defaulted to the answer “all real numbers”. The method of finding the inverse in part (c) was well known and, apart from the range “all real numbers”, the candidate’s answer to part (b) was followed through for the domain in part (c). The order of the functions needed for part (d) was generally well understood and there were many fully correct solutions. The answer $-\sqrt{5}$ was frequently overlooked. Possibly candidates were confused by the range of f , $x > 3$, but as g is applied first, $fg(-\sqrt{5}) = g(7) = \frac{1}{8}$ is a legitimate solution.

Question 6

Most candidates did well in part (a), and in part (b) a wrong domain was the most common loss of a mark. In part (b) an incorrect graph was very common as also was the mark for the **equation** of the asymptote.

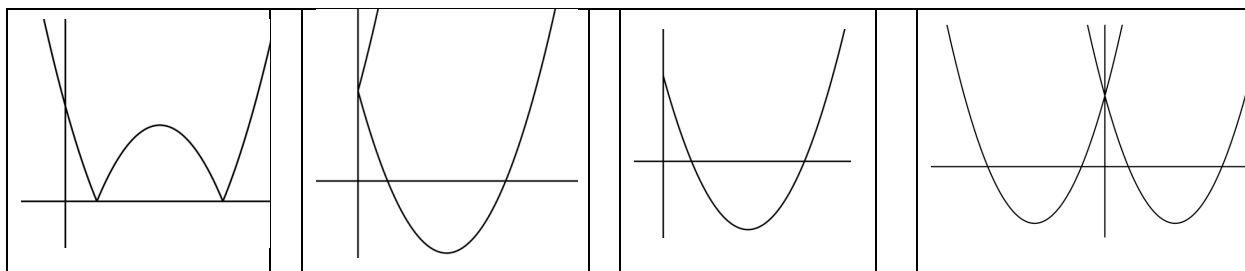
Although part (d) was correctly answered by many, some by using the symmetry of the graph, there was a considerable amount of confusion in finding the second solution, with $\frac{-2}{-x+3} = 3$ and $\frac{2}{-x-3} = 3$ common statements.

Question 7

This question discriminated well across all abilities with about 73% of candidates scoring at least 7 of the 10 marks available, but only about 13% of candidates scoring full marks. The majority of candidates were able to offer fully correct solutions to parts (a) and (b) but sometimes struggled to correctly answer parts (c) and (d).

In part (a), the majority of candidates gained at least 3 out of the 4 marks available. The most common errors were coordinates of $(-3, -4)$ stated in part (i) or $(1.5, -8)$ stated in part (ii).

The majority of candidates gave the correct sketch in part (b), although a few candidates incorrectly gave the coordinates of the turning points or omitted the value where $y = f(|x|)$ meets the y -axis. The four most popular incorrect sketches are shown below.



Only a minority of candidates were able to correctly answer part (c). The most popular correct approach was for candidates to write down an equation for $f(x)$ in the form $(x + p)^2 + q$ by looking at the sketch given in the question. Some candidates incorrectly wrote $f(x)$ as $(x^2 - 3) - 4$, whilst others wrote $y = f(x - 3) - 4$ but could not proceed to the correct answer. The method of writing $f(0) = 5$ and $f(3) = -4$, together with $f(x) = x^2 + ax + b$ in order to find both a and b was successfully executed by only a few candidates.

In part (d), only a minority of candidates were able to explain why the function f did not have an inverse by making reference to the point that $f(x)$ was not one-one or indeed that $f(x)$ was many-one. In some explanations it was unclear to examiners about whether the candidates were referring to the function f or its inverse f^{-1} . The most common examples of incorrect reasons given were ‘you cannot square root a negative number’; ‘the inverse of f is the same as the original function’; ‘it cannot be reflected in the line $y = x$ ’; ‘ f is a one-many function’; or ‘a quadratic does not have an inverse’.

If a candidate made reference to a quadratic function they needed to elaborate further by referring to the domain by saying for example, ‘In f , one y -coordinate has 2 corresponding x -coordinates’.

Question 8

Part (a)(i) was answered very well with a large number of fully correct solutions. The majority of candidates did recognise the need to use the Product Rule, with most wisely quoting it. Some errors were seen in the differentiation of $\ln(3x)$ with the most common mistake being $\frac{1}{3x}$.

In part (a)(ii) the quotient rule provided more room for error than the product rule. Again, wise candidates started by quoting the rule. The majority of candidates who used the quotient rule applied it correctly. The use of the chain rule to differentiate $(2x - 1)^5$ was usually successful, although $5(2x - 1)^4$ was commonly seen. Some candidates did not understand the rules of indices and as a result $((2x - 1)^5)^2$ became $(2x - 1)^7$ or $(2x - 1)^{25}$. This part required the answer to be fully simplified, although this seems to have been missed by some. A significant number were able to cancel out the common factor of $(2x - 1)^4$ and proceed correctly to the final answer. Other errors were seen in the incorrect expansion of brackets.

A minority of candidates attempted the use of the product rule to differentiate. These tended to be less successful. Whilst the use of the product rule for a quotient is perfectly valid, the extra complications involved in simplification tended to lead to a greater number of errors.

In part (b) many candidates were able to achieve the first three marks. It is pleasing to note that the lack of understanding of this part of the specification experienced in previous papers was less evident this year. Most students knew that if $x = 3 \tan 2y$ then $\frac{dx}{dy} = \dots \sec^2 y$. This

was then more often than not correctly followed by $\frac{dy}{dx} = \frac{1}{\dots \sec^2 2y}$. The last part of this question was more demanding. Of those who chose to use the identity “ $\tan^2 2y + 1 = \sec^2 2y$ ”, quite a few candidates struggled with the extra factor of 3 in $x = 3 \tan 2y$.

Statistics for C3 Practice Paper Silver Level S2

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	6		77	4.63		5.42	4.87	4.30	3.77	3.19	2.36
2	9		86	7.77	8.87	8.60	8.21	7.64	6.88	5.84	4.13
3	8		78	6.23	7.80	7.36	6.81	5.94	4.95	3.66	1.96
4	7		78	5.48		6.05	5.34	4.82	4.13	3.45	2.53
5	12		72	8.61		9.96	8.98	8.23	7.35	5.94	3.94
6	12		69	8.24		10.40	8.65	7.42	5.93	4.39	2.51
7	10		67	6.67	9.12	7.93	6.76	5.82	4.95	4.07	2.73
8	11		70	7.67	10.33	9.10	8.04	7.00	5.80	4.48	2.50
	75		74	55.30		64.82	57.66	51.17	43.76	35.02	22.66